Precalculus 1A Semester Credit by Exam Information

Plano ISD Now uses the Texas Tech University ISD Credit by Exam Test for Precalculus

Format: first semester consists of 28 questions; open-response items

The recommended time limit for students to take this exam is 3 hours.

A graphing calculator (TI-84 family) will be provided to you during the exam.

In preparation for the examination, you should review the state standards (TEKS) for precalculus. All TEKS are assessed. Since questions are not taken from any one source, you can prepare by reviewing any resources aligned to these TEKS. For your reference, the instructional materials used in TTUISD are listed below.

Precalculus with Limits: A Graphing Approach (Fourth Edition)
Houghton Mifflin Company, Boston, MA

In order to receive credit for a problem, you must show all of your work and label your graphs. When possible, exact answers should be given. Numerical approximations involving decimals will NOT be accepted for exact answers. You will write your answers on the exam. A percentage score from the examination will be reported to the official at your school.

The practice exam is to help you better prepare for this exam. It is not a duplicate of the actual exam. It is to illustrate the format of the exam, and does not serve as a complete review sheet. Follow the instructions on the practice exam so that you will know what is expected of you when you take the real exam.

Texas Essential Knowledge and Skills
PRE CALC 1A – Precalculus, First Semester

<table>
<thead>
<tr>
<th>TTU: Pre Calc 1A CBE, v.3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEKS: §111.42. Precalculus, Adopted 2012 (One-Half Credit)</td>
</tr>
<tr>
<td>TEKS Covered</td>
</tr>
<tr>
<td>(a) The provisions of §§111.39-111.45 of this subchapter shall be implemented by school districts.</td>
</tr>
<tr>
<td>(b) No later than June 30, 2015, the commissioner of education shall determine whether instructional materials funding has been made available to Texas public schools for materials that cover the essential knowledge and skills for mathematics as adopted in §§111.39-111.45 of this subchapter.</td>
</tr>
<tr>
<td>(c) If the commissioner makes the determination that instructional materials funding has been made available under subsection (b) of this section, §§111.39-111.45 of this subchapter shall be implemented beginning with the 2015-2016 school year and apply to the 2015-2016 and subsequent school years.</td>
</tr>
<tr>
<td>(d) If the commissioner does not make the determination that instructional materials funding has been made available under subsection (b) of this section, the commissioner shall determine no later than June 30 of each subsequent school year whether instructional materials funding has been made available. If the commissioner determines that instructional materials funding has been made available, the commissioner shall notify the State Board of Education and school districts that §§111.39-111.45 of this subchapter shall be implemented for the following school year.</td>
</tr>
<tr>
<td>(e) Sections 111.31-111.37 of this subchapter shall be superseded by the implementation of §§111.38-111.45 under this section.</td>
</tr>
<tr>
<td>Source: The provisions of this §111.38 adopted to be effective September 10, 2012, 37 TexReg 7109.</td>
</tr>
<tr>
<td>§111.42. Precalculus, Adopted 2012.</td>
</tr>
<tr>
<td>(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Algebra I, Geometry, and Algebra II.</td>
</tr>
<tr>
<td>(b) Introduction.</td>
</tr>
</tbody>
</table>

Updated 8/9/2016
(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.

(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(3) Precalculus is the preparation for calculus. The course approaches topics from a function point of view, where appropriate, and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of Precalculus deepens students' mathematical understanding and fluency with algebra and trigonometry and extends their ability to make connections and apply concepts and procedures at higher levels. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology to build understanding, make connections between representations, and provide support in solving problems.

(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

(c) Knowledge and skills.

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace;

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Functions. The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

(A) use the composition of two functions to model and solve real-world problems;

(B) demonstrate that function composition is not always commutative;

(C) represent a given function as a composite function of two or more functions;

(D) describe symmetry of graphs of even and odd functions;

(E) determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;

(F) graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;

(G) graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including af(x), f(x) + d, f(x - c), f(bx) for specific values of a, b, c, and d, in mathematical and real-world problems;

(H) graph arscin x and arccos x and describe the limitations on the domain;

(I) determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;

(J) analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;

(K) analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;

(L) determine various types of discontinuities in the interval (−∞, ∞) as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;

(M) describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities;

(N) analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems;

(O) develop and use a sinusoidal function that models a situation in mathematical and real-world problems; and

(P) determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

Updated 8/9/2016
### (3) Relations and geometric reasoning.
The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:

- **(A)** graph a set of parametric equations;
- **(B)** convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;
- **(C)** use parametric equations to model and solve mathematical and real-world problems;
- **(D)** graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates;
- **(E)** graph polar equations by plotting points and using technology;
- **(F)** determine the conic section formed when a plane intersects a double-napped cone;
- **(G)** make connections between the locus definition of conic sections and their equations in rectangular coordinates;
- **(H)** use the characteristics of an ellipse to write the equation of an ellipse with center \((h, k)\); and
- **(I)** use the characteristics of a hyperbola to write the equation of a hyperbola with center \((h, k)\).

### (4) Number and measure.
The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:

- **(A)** determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems;
- **(B)** describe the relationship between degree and radian measure on the unit circle;
- **(C)** represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;
- **(D)** represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;
- **(E)** determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;
- **(F)** use trigonometry in mathematical and real-world problems, including directional bearing;
- **(G)** use the Law of Sines in mathematical and real-world problems;
- **(H)** use the Law of Cosines in mathematical and real-world problems;
- **(I)** use vectors to model situations involving magnitude and direction;
- **(J)** represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically; and
- **(K)** apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.

### (5) Algebraic reasoning.
The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:

- **(A)** evaluate finite sums and geometric series, when possible, written in sigma notation;
- **(B)** represent arithmetic sequences and geometric sequences using recursive formulas;
- **(C)** calculate the nth term and the nth partial sum of an arithmetic series in mathematical and real-world problems;
- **(D)** represent arithmetic series and geometric series using sigma notation;
- **(E)** calculate the nth term of a geometric series, the nth partial sum of a geometric series, and sum of an infinite geometric series when it exists;
- **(F)** apply the Binomial Theorem for the expansion of \((a + b)^n\) in powers of \(a\) and \(b\) for a positive integer \(n\), where \(a\) and \(b\) are any numbers;
- **(G)** use the properties of logarithms to evaluate or transform logarithmic expressions;
- **(H)** generate and solve logarithmic equations in mathematical and real-world problems;
- **(I)** generate and solve exponential equations in mathematical and real-world problems;
- **(J)** solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems;
- **(K)** solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems;
- **(L)** solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems;
- **(M)** use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and
- **(N)** generate and solve trigonometric equations in mathematical and real-world problems.

*Source: The provisions of this §111.42 adopted to be effective September 10, 2012, 37 TexReg 7109.*
PRE CALC 1A Formula Chart

\[ a_n = a_1 + (n-1)d \]

\[ a_n = a_1 r^{n-1} \]

\[ S_n = \frac{n}{2} (a_1 + a_n) \]

\[ S_n = \frac{a_1 (1 - r^n)}{(1-r)} \]

\[ S = \frac{a_1}{(1-r)} \]

\[ A = P \left(1 + \frac{r}{n}\right)^n \]

\[ A = Pe^{rt} \]

\[ y = Ce^{kt} \]

\[ A = A_0 \left(b^t\right) \]
Directions: Take this exam without reference to any books or notes, exactly as you will during the actual exam. You may use a graphing calculator on the exam, but in order to receive credit for a problem, you must show all of your work and label your graphs. Numerical approximations involving decimals should be written to three decimal places unless otherwise noted. When possible exact answers should be given. (For example: 0.8660 will not be accepted for $\frac{\sqrt{3}}{2}$.)

Write your answers on this exam.

Check your answers with the answer key provided.

1. Use the relationship as described below to answer the following questions.

\[ f(x) = \begin{cases} 
(x + 2)^2 & \text{if } x < 0 \\
4 & \text{if } 0 \leq x < 3 \\
\sqrt{x - 3} & \text{if } x \geq 3 
\end{cases} \]

A. Is \( f \) a function? Justify your answer.  
B. Identify \( x \)- and \( y \)-intercepts if they exist.

C. Find the domain and range of \( f \).  
D. Evaluate \( f(-3) \).

E. Find all values of \( x \) for which \( f(x) = 1 \).
F. Evaluate \( \lim_{x \to 3^-} f(x) \).  
(The limit as \( x \) approaches 3 from the right.)

G. Evaluate \( \lim_{x \to 3^+} f(x) \).  
(The limit as \( x \) approaches 3 from the left.)

H. Evaluate \( \lim_{x \to 3} f(x) \).

I. Is \( f \) continuous at \( x = 3 \)?  
Justify your answer.

2. Given the piecewise defined function \( g(x) \) described below:

\[
g(x) = \begin{cases} 
-x - 1 & x < 0 \\
x^2 - 1 & 0 \leq x \leq 2 \\
-\sqrt{x - 2} + 2 & x > 2 
\end{cases}
\]

A. Graph the function on the axes provided.

B. Evaluate \( g(2) \).
C. Is the function continuous or discontinuous? If the function is discontinuous, identify the \( x \)-value(s) where the discontinuity or discontinuities occur and identify the type of discontinuity.

3. Use parent functions and transformations to provide the equations of the graphs below.

A. \hspace{2cm} B.

Equation: \( y = \) \hspace{2cm} Equation: \( y = \)
4. Given the rational function: \( f(x) = \frac{x^2 - 1}{x^2 - x - 2} \), determine the following.
If the answer is none, please state so.

A. domain =

B. range =

C. \( x \)-intercepts =

D. \( y \)-intercepts =

E. horizontal asymptotes =

F. vertical asymptotes =

G. removable discontinuities =

\(continued \rightarrow\)
5. The number $N$ of bacteria in a refrigerated food is given by:

$$N(T) = 10T^2 - 20T + 600, \quad 2 \leq T \leq 20$$

where $T$ is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by:

$$T(t) = 3t + 2, \quad 0 \leq t \leq 6$$

where $t$ is the time in hours. Find $(N \circ T)(t)$ and interpret its meaning.
6. Find two functions \( f \) and \( g \) such that \((f \circ g)(x) = h(x)\) if \( h(x) = \frac{\sqrt{8-x}}{5} \).

7. Given \( f(x) = \sqrt{2x-3} \)
   
   A. Graph \( f \) and \( f^{-1} \) on the axes provided.

   B. How are the graphs related?

   C. Write an equation for \( f^{-1}(x) \).

continued →
8. Given \( f(x) = \frac{x - 4}{5} \) and \( g(x) = \frac{5}{x - 4} \), find \( f(g(x)) \) and \( g(f(x)) \).

Are \( f \) and \( g \) inverse functions? Justify your answer.

9. Determine whether \( f(x) = x^3 - x \) is even, odd, or neither. Then describe the symmetry.
10. Find all real and imaginary roots of $P(x)$.
$P(x) = x^3 - 3x^2 + 2x - 6$.

11. Use the polynomial function $P(x) = x^3 + 3x^2 - 1$ to answer the following if they exist.

A. relative minimum(s) =

B. relative maximum(s) =

C. intervals where $P(x)$ is increasing =

continued →
D. intervals where $P(x)$ is decreasing =

E. Use the information above to provide a sketch of the function. Label the critical values.

12. Solve: $3x^3 - 4x^2 - 12x > -16$. Express your answer in interval notation.
13. Solve: \( \frac{3x-5}{x-3} \leq 1 \). Express your answer in interval notation.

14. Describe the end behavior of the function shown below.
15. Express as a single logarithm: \( \log_3 x - \frac{1}{5} \log_3 y - 2 \log_3 w \).

16. Express in terms of logarithms of \(a\), \(b\), and \(c\): \( \log_2 \frac{a^{\frac{3}{4}} \sqrt{c}}{b^3} \).

Solve for \(x\). Write the exact answer and the decimal approximation to 3 decimal places.

17. \( 9^{x+1} = 27^{3x-1} \)

18. \( \log_2 (4x + 6) = 4 \)
21. Suppose that $2,500 is invested at an interest rate of 6.3%. How much is the investment worth after 5 years if interest is compounded…

A. quarterly?

B. continuously?

C. How long will it take for the investment to become $4,000 if interest is compounded continuously?
22. The half-life of $^{14}$C is 5,715 years. If 6.5 grams is present now, how much will be present in 1,000 years? (Round your answer to three decimal places.)

23. Evaluate: $\sum_{k=1}^{1}(2k^2 - 1)$.

24. Given the sequence: 1, 5, 9, 13, …
   
   A. Determine if the sequence is arithmetic, geometric, or neither. Justify your conclusion.
B. Find the 40th term of the sequence.

C. Write the series 1 + 5 + 9 + 13 + … + 117 using Sigma Notation and evaluate the sum.

25. Find the sum of the infinite series if it exists.
   \[ 16 + 8 + 4 + 2 + \ldots \]

26. Consider the sequence 1.8, –3.6, 7.2, –14.4, …
   A. What type of sequence is it? Justify your answer.

   B. Find the 25th term of the sequence.

   continued \rightarrow
C. Find the sum of the first 25 terms of the sequence.

27. Write the sequence using a recursive formula.
   6, 7.2, 8.64, 10.368 …

28. Expand the binomial. \((3a - 2b)^6\)
1. A. Yes, $f$ is a function because for every $x$, there is one and only one $y$. \textit{(f passes the vertical line test.)}

   B. $x$-intercepts: $(-2, 0)$ and $(3, 0)$; $y$-intercept $(0, 4)$

   C. Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

   D. 1

   E. $x = -3, x = -1, x = 4$

   F. 0

   G. 4

   H. The limit does not exist.

   I. $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$

   No, the limit does not exist.

   \textit{(Note: There is a hole at $(3, 4)$ on the graph.)}

2. A. \textit{[Diagram of a graph with a hole at $(2, 2)$]}

   \textit{(Note: There is a hole at $(2, 2)$.)}

   B. 3

   C. discontinuous at $x = 2$; jump discontinuity

3. A. $y = [x] - 5$

   B. $y = 2^x$
4. \( f(x) = \frac{x^2 - 1}{x^2 - x - 2} = \frac{(x+1)(x-1)}{(x+1)(x-2)} = \frac{x-1}{x-2} \)

A. Domain: \((-\infty, -1) \cup (-1, 2) \cup (2, \infty)\)

B. Range: \((-\infty, 1) \cup (1, \infty)\)

C. \( x - 1 = 0; x = 1; (1, 0) \)

D. \( y = \frac{1}{2}; \left[ 0, \frac{1}{2} \right] \)

E. \( y = 1 \)

F. \( x = 2 \)

G. \( x = -1 \)

H. (Note: There is a hole at \( x = -1 \); the function is undefined there.)

5. \( N(T(t)) = 10(3t + 2)^2 - 20(3t + 2) + 600 \)
   \[= 10(9t^2 + 12t + 4) - 60t - 40 + 600\]
   \[= 90t^2 + 120t + 40 - 60t - 40 + 600\]
   \[= 90t^2 + 60t + 600\]

\((N \circ T)(t)\) or \(N(T(t))\) represents the number of bacteria present at time, \(t\).

6. \( f(x) = \frac{x}{5}; g(x) = \sqrt{8-x} \)

continued \(\rightarrow\)
7. A. \[ 2x - 3 = 0 \]
\[ 2x = 3 \]
\[ x = \frac{3}{2} \]

B. The graphs are symmetric about the line \( y = x \).

C. \[ y = \sqrt{2x - 3} \]
\[ x = \sqrt{2y - 3} \]
\[ x^2 = 2y - 3 \]
\[ \frac{x^2 + 3}{2} = y; \quad x \geq 0 \]

8. \[ f \left( g(x) \right) = f \left( \frac{5}{x-4} \right) \]
\[ = \frac{5 - 4x + 16}{5(x-4)} \]
\[ = \frac{21 - 4x}{5(x-4)} \]

\[ g \left( f(x) \right) = g \left( \frac{x-4}{5} \right) \]
\[ = \frac{5 - 4 \left( \frac{x-4}{5} \right)}{5} \]
\[ = \frac{25}{x-24} \]

\[ f \text{ and } g \text{ are not inverse functions because } f \left( g(x) \right) \neq x \neq g \left( f(x) \right). \]

continued →
9. \[ f(-x) = (-x)^3 - (-x) \]
   \[ = -x^3 + x \]
   The \( f \) is odd and symmetric to the origin.

10. \[
\begin{array}{cccc}
\text{x} & 1 & -3 & 2 \\
\text{y} & 3 & 0 & 6 \\
\end{array}
\]

\[ x = 3, x = i\sqrt{2} \]

\[ x^2 + 2 = 0 \]
\[ x^2 = -2 \]
\[ x = \pm \sqrt{-2} \]
\[ x = \pm i\sqrt{2} \]

11. A. (0, –1); relative min = –1 at \( x = 0 \).
    B. (–2, 3); relative max = 3 at \( x = –2 \).
    C. increasing: (–\( \infty \), –2) \( \cup \) (0, \( \infty \))
    D. decreasing: (–2, 0)
    E. [Graph Image]

\textit{continued} \rightarrow
12. \(3x^3 - 4x^2 - 12x + 16 > 0\)
\[x = 2 \quad 3x^2 + 2x - 8 = 0\]
\[(3x - 4)(x + 2)\]
\[3x = 4\]
\[x = -2\]
\[x = \frac{4}{3}\]
\[
\begin{array}{cccc}
-2 & + & - & + \\
\hline
\frac{4}{3} & & & 2
\end{array}
\]
\[
\left(-2, \frac{4}{3}\right) \text{ or } (2, \infty)
\]

13. \( \frac{3x - 5}{x - 3} - \frac{x - 3}{x - 3} \leq 0 \)
\[\frac{2x - 2}{x - 3} \leq 0\]
zeros: \(2x = 2\)
\[x = 1\]
undefined: \(x = 3\)
\[
\begin{array}{cccc}
+ & - & + \\
\hline
1 & & & 3
\end{array}
\]
\([1, 3]\)

14. As \(x \to \infty\), \(y \to \infty\); this may be written as \(\lim_{x \to \infty} f(x) = \infty\)
As \(x \to -4\) from the right, \(y \to -\infty\); this may be written as \(\lim_{x \to -4^+} f(x) = -\infty\)

15. \(\log_3 \sqrt[3]{y} \frac{x^6}{w^2}\)
16. \(3\log_2 a + \frac{1}{4}\log_2 c = 5\log_2 b\)

17. \((3^2)^{x+1} = (3^3)^{3x-1}\)
\[2x + 2 = 9x - 3\]
\[5 = 7x\]
\[
\begin{array}{c}
\frac{5}{7} = x \\
\end{array}
\]

18. \(2^4 = 4x + 6\)
\[16 = 4x + 6\]
\[10 = 4x\]
\[2.5 = x\]

19. \(e^{2x} = 5\)
\[
\ln e^{2x} = \ln 5 \\
2x = \ln 5 \\
\]
\[
\begin{array}{c}
x = \frac{\ln 5}{2} = .805 \\
\end{array}
\]

20. \(\log_3 \frac{5x+13}{6} = \log_3 3x\)
\[
\frac{5x+13}{6} = 3x \\
5x + 13 = 18x \\
13 = 13x \\
1 = x
\]

continued →
21. A. \[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]
   \[= 2500 \left(1 + \frac{0.063}{4}\right)^{4(5)}\]
   \[= \$3,417.25 \]

   B. \[ A = Pe^{rt} \]
   \[= 2500e^{0.063(5)}\]
   \[= \$3,425.65 \]

   C. \[ A = Pe^{rt} \]
   \[4000 = 2500e^{0.063t}\]
   \[1.6 = e^{0.063t}\]
   \[7.460 \text{ years} \approx \frac{\ln 1.6}{0.063} = t \]

22. \[ A = A_0(b)^k \]
   \[= 6.5 \left(\frac{1}{2}\right)^{\frac{1000}{5715}}\]
   \[= 5.758 \text{ grams} \]

23. \[ 1 + 7 + 17 + 31 = 56 \]

24. A. The sequence is arithmetic because there is a common difference between consecutive terms. The common difference is 4.

   B. \[ a_n = a_1 + (n-1)d \]
   \[= 1 + (39)(4)\]
   \[= 157 \]

   continued →
C. \[ \sum_{n=1}^{30} 4n - 3 = S_n = \frac{30}{2} (1 + 117) = 1770 \]

\[ a_n = 1 \cdot (n-1)(4) = 4n - 3 \]

\[ 4n - 3 = 117 \]

\[ n = 30 \]

25. Geometric; \( r = \frac{1}{2} \)

\[ S = \frac{a_1}{1 - r} = \frac{16}{1 - \frac{1}{2}} = \frac{16}{\frac{1}{2}} = 32 \]

26. A. The sequence is geometric because there is a common ratio of \(-2\) between consecutive terms.

B. \[ a_n = a_1 \cdot r^{n-1} = 1.8(-2)^{24} = 30,198,988.8 \]

C. \[ S_n = \frac{a_1 \left(1 - r^n\right)}{1 - r} = \frac{1.8 \left(1 - (-2)^{25}\right)}{1 - (-2)} = 20,132,659.8 \]

27. \[ a_1 = 6 \]

\[ a_n = \left[ (a_{n-1})(1.2) \right] \]

\[ continued \rightarrow \]
(3a)⁶ + 6(3a)⁵(−2b) + 15(3a)⁴(−2b)² + 20(3a)³(−2b)³ + 15(3a)²(−2b)⁴ + 6(3a)(−2b)⁵ + (−2b)⁶

729a⁶ - 2916a⁴b² + 4860a²b⁴ - 4320ab⁵ + 2160b⁶ - 576ab⁵ + 64b⁶