

Build Math Justifications to Develop Strong Math Thinkers

Cheri Ladner

cladner@classworks.com

Justifying Justifications!



Construct viable arguments and critique the work of others

(Math Practice Standard #3)

Students must:

- Develop and communicate mathematical knowledge
- Support numerical answers through mathematical principles using evidence
- Reflect conceptual understanding not just correct procedures

Why is this difficult?

- Lack of conceptual understanding
- Lack of exposure to sense-making tasks
- Lack of practice
- Lack of tools to develop justification

Justifying Your Answer in Math

- »» Label your work to make it clear.
- »» Show all of your work. This will be your evidence.
 - »» Models
 - »» Equations
 - »» All steps taken to solve the problem
- »» Explain how you got your answer in writing.
 - »» First I... then I...
 - »» I... then I... Finally I..
- »» Explain why you chose an operation.
 - »» I subtracted/added/multiplied/divided because
- »» Explain how you know your answer is correct
 - »» I know my answer is correct because ...
 - »» My model shows my answer is correct because ...
- »» Show that your answer is correct with multiple representations.
 - »» Use more than one model to show your answer is correct.

Let's try it!



Jacob and Marco have many pens.

Jacob has 4 black pens. Marco has 5 blue pens.

How many more pens do they need in order to have a total of 10 pens?

Show Hint

A 0 more pens

B 4 more pens

C 2 more pens

D 1 more pen

I'm Finished



Jacob has 4 pens, and Marco has 5 pens. First, I added these two amounts together to find the total number of pens the two boys have together.

$$4 + 5 = 9$$

The two boys have 9 pens total. Next, I subtracted the number of pens the boys have from 10 to find the number of pens they still need.

$$10 - 9 = 1$$

Because the boys have 9 pens right now and they need 10 pens total, they need 1 more pen.

I know my answer is correct because my models show that $4 + 5 = 9$, and that $10 - 9 = 1$.

Jacob



Marco

I added 4 and 5 together because I need to know how many pens the boys have altogether.

$$\begin{array}{r} 4 \\ + 5 \\ \hline 9 \end{array}$$

Need



The boys need 1 more pen to have 10 pens.

Have



$$10 - 9 = 1$$

I subtracted because I needed to find out how many more pens the boys needed to get the total. I had to find the difference between 10 and 9.



Jacob and Marco have many pens.

Jacob has 4 black pens. Marco has 5 blue pens.

How many more pens do they need in order to have a total of 10 pens?

Show Hint

A 0 more pens

B 4 more pens

C 2 more pens

D 1 more pen

I'm Finished

Label

S

Jacob has 4 pens, and Marco has 5 pens. First, I added these two amounts together to find the total number of pens the two boys have together.

$$4 + 5 = 9$$

The two boys have 9 pens total. Next, I subtracted the number of pens the boys have from 10 to find the number of pens they still need.

$$10 - 9 = 1$$

Because the boys have 9 pens right now and they need 10 pens total, they need 1 more pen.

I know my answer is correct because my models show that $4 + 5 = 9$, and that $10 - 9 = 1$.

Model

Jacob



Marco

4

5

9

I added 4 and 5 together because I need to know how many pens the boys have altogether.

Steps to solve and Explanation

Need



The boys need 1 more pen to have 10 pens.

Have



Equation

$$10 - 9 = 1$$

I subtracted because I needed to find out how many more pens the boys needed to get the total. I had to find the difference between 10 and 9.



22.2102.13855

Stella and Caleb each count the coins in their piggy banks.

Caleb counted 125 nickels in his bank. This is less than the number of nickels in Stella's bank. Which could be the number of Stella's nickels?

Show Hint

A

125

B

118

C

151

D

102

I'm Finished

If Caleb has 125 nickels, the only answer that could be more than that amount is 151.

I know that 151 is more than 125 because when I am comparing the numbers in their place values, the hundreds is the same, but in the tens place, 5 is more than 2, so 151 is greater than 125.

Which student solved this problem correctly?

Show Hint

A Paul solved the problem like this:

The area of the play space is 54 square meters. I divided the shape into two rectangles and found the area of each.

The tall rectangle on the right is $10 \times 3 = 30$ square meters.

The bottom rectangle is $8 \times 3 = 24$ square meters.

$30 + 24 = 54$ square meters

B Samantha solved the problem like this:

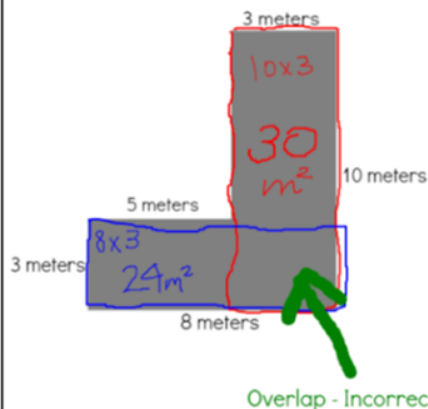
The area of the play space is 45 square meters. I divided the shape into two rectangles and found the area of each.

The top rectangle is $7 \times 3 = 21$ square meters.

The bottom rectangle is $8 \times 3 = 24$ square meters.

$21 + 24 = 45$ square meters total

Use the canvas to explain the mistake made in the incorrect solution.



PAUL'S MISTAKE

The mistake that Paul made was that he overlapped the small square when he was calculating the area of the figure. Because he did this, the area value that he got is greater than the area of the play space for this problem.

PAUL'S METHOD

Paul was correct to divide the shape into two rectangles and to first find the area of both rectangles individually, and then to add the two rectangles together.

Paul divided the shape into a right rectangle, outlined in red on my model, to be 10×3 , with an area of 30 square meters.

Then, Paul divided the shape into a bottom rectangle, outlined in blue on my model, to be 8×3 , with an area of 24 square meters.

The two calculations the Paul did to find the areas of the two rectangles are correct. He used the area formula, length $\times$ width, to calculate the area.

Lastly, Paul added the two area values together to find the area of the whole figure. This calculation is also done correctly.

POSSIBLE CORRECTION

To correct this mistake, if Paul wanted to keep the two rectangles divided as he did instead of like Samantha did, he could find the area of the overlap square and then subtract that from the area he calculated, 54 square meters. This would subtract the space that was counted twice in his original calculations, and would give him the area of the play space.

Area of Overlap Square: $3 \times 3 = 9$

I used the area formula of length $\times$ width to calculate this value.

Area of original calculations minus the area of the overlap square:
 $54 - 9 = 45$

The area of the play space is 45 square meters.



22 2102 13935

Kai and his dad are building a doghouse for their new puppy.

Kai and his dad have four boards. The lengths of the boards are shown.

$\frac{4}{6}$ yard $\frac{8}{12}$ yard $\frac{2}{4}$ yard $\frac{2}{3}$ yard

Which lengths are equivalent? Explain how you know.



Show Hint

I'm Finished



$$\frac{4}{6} \times \frac{2}{2} = \frac{8}{12} \text{ yd}$$



$$\frac{8}{12} \times \frac{1}{1} = \frac{8}{12} \text{ yd}$$



$$\frac{2}{4} \times \frac{3}{3} = \frac{6}{12} \text{ yd}$$

$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12} \text{ yd}$$



$\frac{4}{6}$, $\frac{8}{12}$, and $\frac{2}{3}$ are all equivalent fractions.



22 2102 13827

Jody is comparing the lengths of some objects in her classroom.

Jody compares the heights of three of her schoolbooks. Her math book is taller than her reading book. Her spelling book is taller than her reading book.

What does Jody need to do next in order to put the heights of the three books in order from tallest to shortest?



Show Hint

I'm Finished



22 2102 13829

Many of Billy's friends like to collect and trade seashells.

Samantha had 11 seashells. She gave 2 seashells to Wendy. Write a number sentence to show Samantha's seashells. How many seashells does Samantha have now? Explain your thinking.

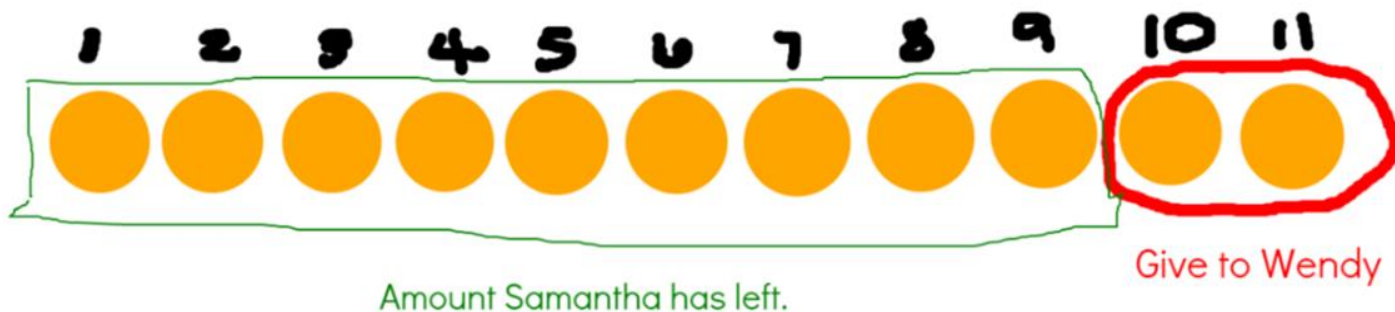
[Show Hint](#)[I'm Finished](#)

Number sentence for this problem: $11 - 2 = 9$

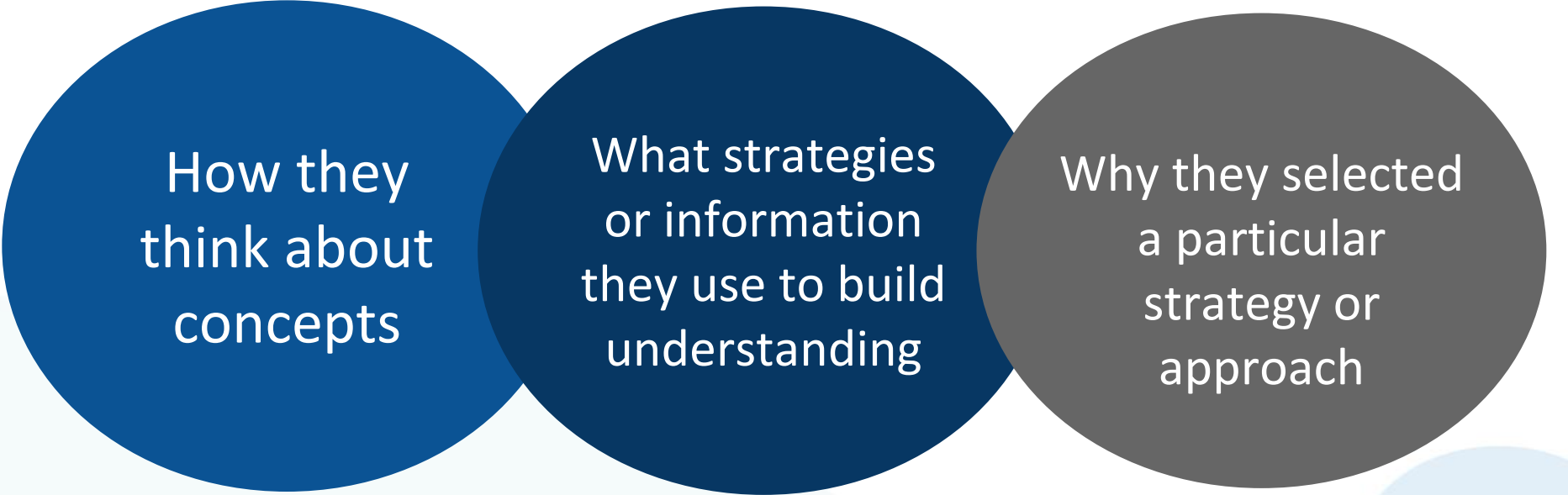
Samantha will have 9 seashells after she gives Wendy 2 seashells.

I subtracted the number of seashells Samantha gave to Wendy from the amount of seashells that Samantha started with to find the number of seashells she will have left.

I chose to subtract 2 from 11 because Samantha is giving these seashells to Wendy, so she will have less than the number she originally had.



Build Critical Thinking: Articulate and Illustrate



How they
think about
concepts

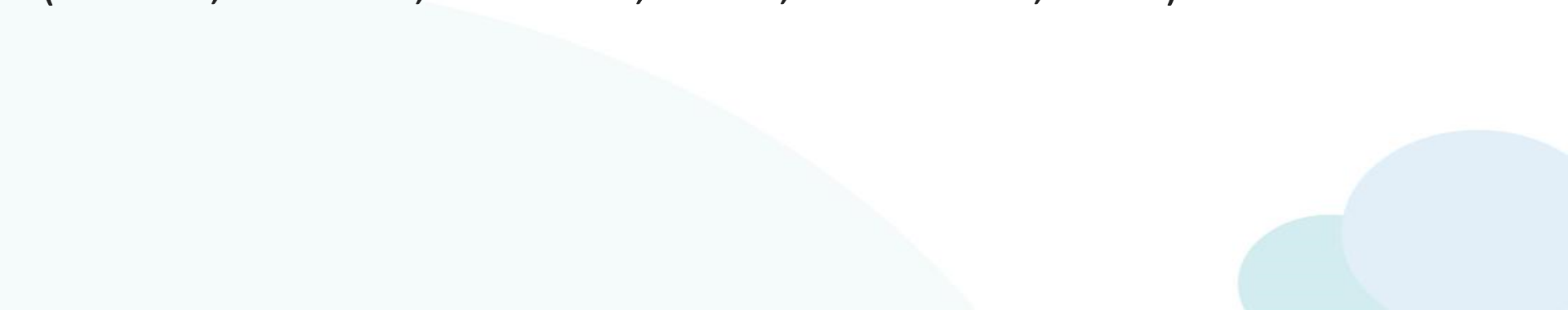
What strategies
or information
they use to build
understanding

Why they selected
a particular
strategy or
approach

Research Tells Us...

Notational systems, specialized vocabulary, and various technological and other tools also free up memory for more complex tasks.

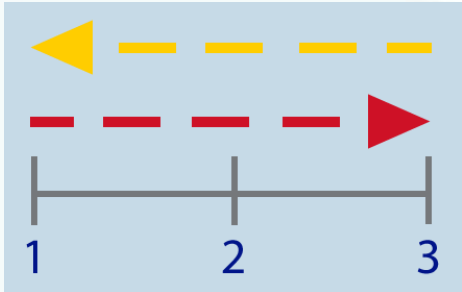
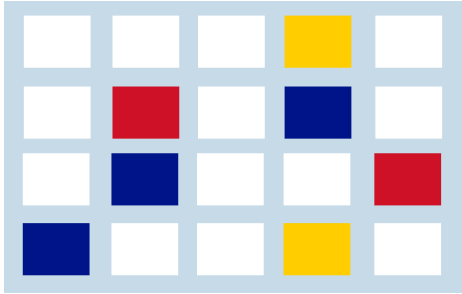
(Perkins, Tishman, Ritchhart, Donis, & Andrade, 2000)



Curriculum that supports students and teachers!

- ✓ Written to have **conceptual scenarios** for real world application
- ✓ Built in **differentiation**
- ✓ **Digital tools** to model and support justifications
- ✓ **Scaffold** math communication with audio recording

Curriculum that supports students and teachers!



- ✓ Digital manipulatives
- ✓ Sample responses
- ✓ Real time feedback
- ✓ Platform for personalized feedback

Resources

“Probing Questions.” Changingminds.org, n.d. Web. 9 Dec. 2015.

http://changingminds.org/techniques/questioning/probing_questions.htm

“Probing Questions to Help Students Think Critically about Reading.” (n.d.): n. pag. www.ramapo.edu, 2013. Web. 10 Dec. 2015.

<http://www.classworks.com/> Classroom reading and math instruction designed to address the depth and rigor required by today’s standards. Built to support teachers during whole group, small group and individual instructional practices.

Sahin, Alpaslan. “The Effects of Types, Quantity, and Quality of Questioning in Improving Students’ Understanding.” (2007). <http://oaktrust.library.tamu.edu/> Web. 9 Dec. 2015.

<http://bit.ly/1O9d2XO>

Franke, Megan L., Webb, N., and Chan, A. “Eliciting Student Thinking in Elementary School Mathematics Classrooms.” (2007): n. pag. <https://www.cse.ucla.edu>. Web. 8 Dec. 2015.

<https://www.cse.ucla.edu/products/reports/R725.pdf>

Findley, Jennifer. “Teaching Students How to Justify Answers in Math.” (2016) *Teaching to Inspire*

<http://teachingtoinspire.com/2016/02/teaching-students-how-to-justify-answers-in-math.html>